I B.Tech Regular Examinations,June 2010
MATHEMATICS-1
Common to ME, CHEM, BME, IT, MECT, MEP, AE, BT, AME, ICE,
E.COMP.E, MMT, ETM, EIE, CSE, ECE, EEE,CE

Time: 3 hours   Max Marks: 75

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Prove that div $(r^n \bar{r}) = (n+3)\, r^n$. Hence show that $\frac{\bar{r}}{r^3}$ is solenoidal
   (b) If $\bar{F} = (x+y+1)\, i + j - (x+y)\, k$, then show that $\bar{F}$ curl $\bar{F}$=0   [7 +8]

2. (a) Form the differential equation by eliminating arbitrary constants $\log \frac{y}{x} = $ C
   (b) Solve the differential equation $\frac{dy}{dx} = e^{2x-3y} + x^2 e^{-3y}$
   (c) If the air is maintained at 30 ^{0}C and the temperature of the body cools from 80 ^{0}C to 60 ^{0}C in 12 minutes, find the temperature of the body after 24 minutes.   [4+5+6]

3. (a) Find $L\left[e^{-3t} \sinh 3t \right]$ using change of scale property.
   (b) Solve the following differential equation using the Laplace transforms
   $y^{11}\_ 3y^1 + 2y = 4t + e^{3t}$, $y(0) = y^1\,(0) = 1$.   [8+7]

4. (a) Test the convergence of the series $\frac{3^2}{6^2} + \frac{3^2 . 5^2}{6^2 . 8^2} + \frac{3^2 . 5^2 . 7^2}{6^2 . 8^2 . 10^2} + \dots$
   (b) Find the interval of convergence for the following series $\frac{\sum (-1)^n (x+2)}{(2^n + 5)}$ [7+8]

5. (a) If ρ_1 and ρ_2 are radii of curvature at the extremities of any chord of the cardioids $r = a\,(1 + \cos\theta)$, which passes through the pole, then show that $\rho_1^2 + \rho_2^2 = \frac{16a^2}{9}$
   (b) Find the envelope of $\frac{x}{a} + \frac{y}{b} = 1$. Where $a^2 + b^2 = c^2$ [8+7]

6. (a) Expand $e^{x \sin x}$ in powers of x.
   (b) Find the volume of the greatest rectangular parallelopiped that can be 'inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.   [8+7]

7. (a) Find the Volume of the solid obtained by revolving one loop of the curve $r^2 = a^2 \cos^2 \theta$ about the line $\theta = {}^{\pi}/_2$.
   (b) Evaluate by changing the order of integration $\int_0^a \int_{\sqrt{ax}}^a \frac{y^2 dy dx}{\sqrt{y^4 - a^2 x^2}}$ [7+8]

8. (a) Solve the differential equation $(D^2 - 4)y = 2\cos^2 x$
   (b) A particle is executing S.H.M, with amplitude 5 meters and time 4 seconds. Find the time required by the particle in passing between points which are at distances 4 and 2 meters from the centre of force and are on the same side of it.   [8+7]

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